AL-UMAWĪ AL-ANDALUSĪ AND HIS CONTRIBUTION TO ARITHMETIC

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Man, from the very beginning, even before the existence of written language divulged in mathematics without assigning its name to it. He tried to count the objects, add, subtract and divide them as per his need; introduced the numbers and made efforts to solve his major and minor problems. With the passage of time, the field was evolved gradually with new formulae invented and rules chalked out by different civilizations, nations, races and schools. The origin of mathematics can not be traced out but possibly it was the Babylonian[1] among the earliest civilizations that contributed first to the field. Sargeon, the ruler of Babylonia, about 2700 B.C. showed considerable interest in mathematical learning and patronized its scholars.

From that period onwards different civilizations produced a number of great mathematical scholars. Greeks, Thales (640-546 B.C.), Pythagoras (569-500 B.C.), Plato (429-340 B.C.) and Ptolemy were great mathematicians who established the Ionian School, the Pythagorian School, the Platonic School and the first Alexandrian School respectively. Civilizations that can be considered to be the major contributors to mathematics are: The Egyptian, the Greek, the Roman, the Maya, the Chinese, the Japanese, the Indian or the Hindu and the Arab. It is markable that no civilization borrowed the numerals of the others, but introduced and used its own numeral system with each showing its own interest in different branches of mathematics. The Babylonians worked especially on sixagesinal numbers and also made two tablets giving the numbers in square upto sixty and the illuminated portion of moon's disc of every day from new to full moon respectively. The Egyptians and the Greeks concentrated more on geometry than on other branches of mathematics. The Hindus developed arithmetic while the Arabs made all branches as their subject of attention, particularly algebra and trigonometry.

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[1] It was the civilization flourished in the valley between Tigris and Euphrates between 3100 and 2100 B.C.

Muslim Spain, like other Islamic centres served as great contirubutor to all forms of knowledge. Every field is credited to produce such great scholars whose familiarity was not confined to Spain only but Muslim East as well as Christian West recognised them well. It was regarded as the second high ranked Muslim state after Baghdad.

As with other subjects the scholars gave primary significance to mathematics. Along with religious sciences mathematics was given top priority because of its importance not only in daily life but also in the administration of religious affairs as well such as *Zakāt* and *farā'id* (inheritance law) etc. The works of Spainish Muslim mathematicians were taught in North African schools, those in the East and the Christian West apart from Spanish schools, even upto sixteenth/seventeenth century A.D.

Abū ʿAbdullah Yaʿīsh ibn Ibrāhīm ibn Yūsuf ibn Simāk al-Umawī was a Spanish-Muslim mathematician who devoted his full time and energies to arithmetic alone while others worked on subjects, such as astronomy, astrology, medicine, surgery, pharmacy, music, logic, *Qur'ān, Hadīth, Fiqh, Tafsīr*, alchemy etc. in addition to mathematics.

Abū'l-Qāsim Maslamah al-Majrīṭī (d. 1007 A.D.) along with a number of works on different branches of mathematics such as commercial arithmetic (sales, valuation and taxation)[2], mensuration, amicable numbers and Menelaus and al-Battanī's theorems, worked on astronomy and alchemy, who for his mathematical works, was given the title *al-ḥāsib* (the mathematician)[3] and imām al-riyāḍiyyīn (the leader of mathematicians).

Abū 'Abdullāh Muḥammad ibn Mu'ādh al-Jayyāni (989/990-1079 A.D.), a native of Jean was another well-known mathematician. Besides his valuable works on geometry and spherical trigonometry, wherein he disclosed the angle as the fourth magnitude along with the 'body', 'surface' and 'line'[4], defined five magnitudes used in geometry and determined the magnitudes of the arcs on the surface of a sphere; he devoted a considerable period of his life to Arabic philology, *farā'iḍ* (inheritance law), the study of Quran and especially to astronomy. It may not be out of place to mention here that he wrote a number of astronomical works giving the

[2] J. Vernet, 1986 A.D., *Encyclopaedia of Islam*, (New Edition) E.J. Brill, London, Vol. V, p. 1109.

[3] P.K. Hitti, 1939 A.D., *History of the Arabs*, The Macmillan Press Ltd., London, p. 570.

[4] Cf. Ibn Rushd's *Tafseer* (II. P. 665) by Y.D. Samplonius and N. Hermelink. 1973 A.D. *Dictionary of Scientific Biography*, Edited under the auspices of American Council of Learned Societies, Charles Seribner's Sons, New York, vol. VII., p. 82.

angle of dippression of morning and evening twilight as 18°, and instruction about the direction of meridian, describing about the visibility of new moon, the time of the day, the time and direction of prayer, mentioning about the setting up of horoscope and the prediction of eclipses.

Jābir ibn Aflaḥ, a native of Seville (Ar. Ishbīlia) and a twelfth century mathematician, wrote a marvellous work on Ptolemy's *Almagest* in nine volumes; commenting at different places, supporting some of its statements, arguments and formulas and opposing and correcting those which he considered wrong. This work of Jābir came to be known under two different titles; *Iṣlāḥ al-Majisṭī* and *Kitāb al-Hai'ā*, discussing both mathematics (especially trigonometry) and astronomy. It, with its translation particularly into Latin and Hebrew by Gerard of Cremona and Moses ibn Tibbon, respectively exercised a tremendous influence upon the West especially due to its use by various scholars into their works and its teaching at almost all learning centres of Europe till the seventeenth century A.D. Jābir obviously seems to be different from others as his whole work is based on theory rather than on numerical examples.[5]

Muḥyi al-Dīn al-Maghribī, al-Zarqālī and al-Qalaṣādī are other eminent mathematicians of Muslim Spain. Who put their efforts on other subjects besides mathematics.

Different reports regarding the flourishing period of al-Umawī are available. Brockelmann[6] and A.S. Saidan[7] unanimously mention that he died in 895 A.H./1489 A.D. Other reports suggest that 'Abdul-Qādir al-Maqdisī, a copyist of his work on arithmetic was given the licence to teach his work of nine folios on 17th Dhū'l-Ḥijja, 774 A.H./9th June 1373 A.D. indicating thereby that Umawī florished in or before 1373 A.D. He migrated from Spain and lived for a long period at Damascus where he taught a good number of students.

Al-Umawī worked on different sections of arithmetic viz. fractions, mensuration, square-roots, cube-roots, sequences and series of polygonal and pyramidal numbers, arithmetic and geometric progressions, number theory and summation of r^3, $(2r+1)^3$, $(2r)^3$, $r(r+1)$, $(2r+1)$, $(2r+3)$,

[5] R.P. Lorch, *Dictionary of Scientific Biography*, Vol. VII, p. 38.

[6] C. Brockelmann. 1938 A.D., *Geschichte der Arabischen Literature*, E.J. Brill, Leiden, Zweiter Supplement and II. p. 344.

[7] Cf. Hājji Khalīfah, Kashf al-Dhunūn by A.S. Saidan 1976 A.D., *Dictionary of Scientific Biography*, Edited under the auspices of American Council of Learned Societies. Charles Scribner's Sons, New York, Vol. XIII, p. 539.

$2r(2r+2)$ from $r=1$ to n.[8] He also paid special attention to 'the sum of natural numbers', 'natural odd and even numbers' as well as in geometrical progression $2r$ and $\sum_{r=0}^{n} 2r$. Earlier the sequence of polygonal and pyramidal numbers were transmitted by Thābit ibn Qurra' (836-901 A.D.) from the work of Nicomachus,[9] *Introduction to Arithmetic* and geometrical proof of $\Sigma\, r^3$, $(2r+1)^3$, $(2r)^3$ was given by al-Kharajī. Al-Umawī, used no numerals except in the tables of sequences.

Al-Umawī invented and described briefly a number of methods related to addition, subtraction and multiplication while dealing with Indian arithmetic. In addition he used the summation of sequences and in this connection the chose the path made by Babylonians, followed by Diophantus[10] and Arab mathematicians. He belonged to that category of mathematicians who used no symbol in addition and adopted the trend to take the sum of ten terms as an example. While dealing with subtraction, he followed the Hindu and the Arab mathematicians in castingout[11] the numbers. But unlike the Hindus and the Arabs who casted out nines alone, he casted out sevens, eight, nines and elevens. According to him, if P is any integer after casting out P's from N the remainder of 10^s is γ_s [i.e. $10^s = r_s$ (mod P.)] and $\Sigma\, a_s \times r_s$ is divisible both by P and N. He, in this example considered casting out sevens, eights and elevens and derived the rule

Where N is given as any integer in any decimal scale. $N = a_0 + a_1 x10 + a_2 x10^2 + a_3 x10^3 + \ldots = a_s x10^s$. So when 7 is taken as the value of P then r_s (1,3,2,6,4,5).

At many places al-Umawi does not agree with the mathematicians of the East. He developed the rules of approximation as:

$$\sqrt{n} = a + \frac{n-a^2}{2a} \quad \text{or } (a+1) - \frac{(a+1)^2-n}{2\,(a+1)}$$

$$\sqrt[3]{n} = a + \frac{n-a^3}{2a} \quad \text{or } (a+1) - \frac{(a+1)^3-n}{3\,(a+1)^2}$$

[8] *Ibid.*

[9] He was a great mathematician of the first Alexandrian School.

[10] Diophantus was also an important mathematician of the first Alexandrian School.

[11] The act of obtaining the remainder when a given natural number is divided by an integer n, is known as castingout n's.

while the rule were introduced by the Eastern Scholars as:

$$\sqrt{n} = a + \frac{n-a^2}{2a+1}$$

$$\sqrt[3]{n} = a + \frac{n-a^3}{3a^2+3a+1}$$

Here a^2 and a^3 are the greatest integral square and cube in n. These rules of al-Umawī, according to Saidan, "... are noι well developed as those of the arithmeticians of the East."[12] He also introduced a number of rules for finding perfect square and cube which came to be known in the east only through his own efforts. With the supposition of n as a perfect square he disclosed the rules as:[13]

1. The units' digit of every number should either be 1,4,5,6,9 or the number ends with even zeros (i.e. two zeros, four zeros, six zeros,...)
2. The tens' place should be odd in case that units' place is 6 otherwise it should be even.
3. If the number has its units' place as 1, then the hundreds' place and half of the tens' place should be both even or both odd.
4. For every number, units' place of which is 5, its tens' place should be 2.
5. $$n \equiv 0,1,2,4, \pmod 7$$ $$\equiv 0,1,4 \pmod 8$$ $$\equiv 0,1,4,7 \pmod 9$$

Similarly, for perfect cube n, he presented the following:

1. If the last digists of the numbers are 0,1,4,5,6 and 9 then their cube roots end with 000, 1,4,5,6 and 9 respectively. Also if the last digits of the numbers are 3,7,2 and 8 then the cube roots should end with 7,3,8 and 2 respectively.

2. $$n \equiv 0,1,6 \pmod 7$$ $$\equiv 0,1,3,5,7 \pmod 8$$ $$\equiv 0,1,8 \pmod 9$$

[12] A.S. Saidan, *op. cit.*, p. 540.
[13] *Ibid.*

Al-Umawī, like some other Spanish-Muslim mathematicians as al-Qalaṣādī (1412-1486 A.D.)[14] opposed the Eastern Muslim as well Indian mathematicians regarding fractions who used write them as $\frac{a}{b}$ or $\overset{o}{\underset{b}{a}}$. He separated the numerator and denominator with a line between them. He also used to underline, the numbers to separate them from the steps of operation.

Al-Umawī, discussing the above mentioned arithmetical fields, wrote a number of books and treatises of which only the following are available:

1. *Raf al-Ishkāl fī Masāhat al-Ashkāl* (removal of doubts concerning the mensuration of figures), a treatise on mensuration in 17 folios;
2. *R. fī 'Ilm al-Qabbān;* and
3. *Marāsim al-Intisāb fī 'Ilm al-Ḥisāb.*

The last work of al-Umawī, *Marāsim* is considered to be an important work in 18 folios which exercised great influence upon the west and written according to Saidan, "....by a Western Muslim for Easterners."[15] In this work he modifies the Indian method of calculation by 'dust board' that also contains 'counting by fingers and Pythagorian theory of numbers. He, apart from other works, described in this book the history of numbers also.

With his important works, al-Umawī got familiarity not only in the whole Spanish peninsula but also in other parts of Europe, North Africa as well as in the East. It is due to his ability and excellence in the field that his pupils are found in Spain as well so far distant places from it as Syria.

[14] A.S. Saidan *op. cit.*, Vol. XI, p. 229.
[15] A.S. Saidan, *op. cit.*, Vol. XIII, p. 539.